# Mean-field theory of fluid neural networks

Jordi Delgado[1,2] and Ricard V. Solé[2]
[1]*Departament de Llenguatges i Sistemes Informatics, Universitat Politecnica de Catalunya, Campus Nord, Mòdul C6, Jordi Girona Salgado 1-3 08034 Barcelona, Spain*
[2]*Complex Systems Research Group, Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Sor Eulàlia d' Anzizu s/n, Campus Nord, Mòdul B4, 08034 Barcelona, Spain*
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Fluid neural networks (FNN) are a mathematical framework where the phenomenon of self-synchronization in ant colonies can be explained, predicting the model a critical density, i.e., a density where oscillations appear, observed in real ant colonies. However, up to now all results have been solely numerical. In this paper we put forward a simple FNN with the same phenomenology as the original one, but an analytical approximation can be performed in such a way that critical densities can be computed, offering a good approximation to the numerical ones. [S1063-651X(98)14602-0]

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## I. INTRODUCTION

Collective behavior in ant colonies is undoubtedly a fascinating subject. Hundreds or thousands of small *simple* insects display coordinated collective patterns of complex behaviors, such as raid patterns, food foraging, brood care, task allocation, or nest building, showing how, generally speaking, simple *interacting* individuals can behave as a whole in unexpected ways. Swarm behavior has attracted the attention of physicists working on critical phase transitions or dynamical systems. It has been shown that behind some phenomena such as trail patterns [1] or self-synchronization of activity [2] there are mechanisms well known to physicists, where noise-induced transitions [3] or marginal stability [4] are but a few examples. Beyond the classical interest in swarm behavior from biologists [5], complexity sciences have taken ant colonies as one of the main subjects of study in their quest for laws behind complex phenomena [6].

The mechanisms underlying swarm intelligence, as it is also called, are certainly not few in number though we will be interested mainly in mechanisms by which the global performance of the colony goes beyond that of individuals, such as interactions by means of laying pheromones or by physical contact among individuals. These different ways of interaction may generate striking behaviors, such as stigmery (a stigmergic process, following Wilson [7], is a process by which it is the work already accomplished that induces the insects to perform additional labor) or the one we will focus on in this paper: self-synchronization.

Some experiments with *Leptothorax acervorum* ants by Franks [8] and *Leptothorax allardycei* by Cole [2] revealed the existence of short-term rhythms of activity. This synchronization in activity seems to be especially apparent in nurse workers, where cycles of approximately 20 min (15 min of quiescence plus 2–5 min of activity) have been measured. There exist some mathematical models of this behavior in very different frameworks: differential equations [9], probabilistic process algebra [10] or fluid neural networks (FNN) [11], though up to now it is far from clear which one is best fitted to the phenomenon under study. The interest in self-synchronization is not solely a biological one, since this phenomenon has been shown to be related to mutual exclusion in brood care [12] and task allocation [13]. Thus, the possible usefulness of self-synchronization in ant colonies makes it also interesting for distributed asynchronous algorithm designers.

In this paper we offer an analytical approximation to FNNs, one of the mathematical models of self-synchronization in ant colonies [14]. After reviewing the original FNN in Sec. II, in Sec. III we introduce and justify what we believe is a simpler FNN, with the same phenomenology as the original one, where analytical approximations can be made in order to obtain critical densities near to the ones computed numerically.

## II. FLUID NEURAL NETWORKS

FNNs are defined as formal neurons [15] moving on a lattice. Each ''neuron ant'' has a continuous state $S_i(t) \in \mathbb{R}$, at each time step $t$. Interaction with nearest individuals, located in the neighborhood $B(i)$ defined by the eight nearest lattice sites, is defined by

$$S_i(t+1)=\Phi\left[ g\left\{ J_{ii}S_i(t)+\sum_{i\neq j\in B(i)} J_{ij}S_j(t)-\Theta_i \right\}\right],$$

where $J_{ii}\neq 0$. For simplicity we use the threshold $\Theta_i=0$, and we take $\Phi(z)=\tanh(gz)$ where $g$ is a gain parameter. Each automaton can be either *active* or *inactive*, depending on the state $S_i(t)$ and, if active, it moves randomly to one of the eight nearest cells (if no space is available, no movement takes place). In FNN a given automaton will be active if it is above some threshold $\theta_{\rm act}$, $S_i(t)>\theta_{\rm act}$, and inactive otherwise. Once an automaton becomes inactive, it can return to the active state (with a *spontaneous activity level* $S_a$) with some probability $p_a$. The coupling matrix $\mathbf{J}$ is not fixed. Connections are local and changing over time as a consequence of movement. They are also state dependent, i.e., $J_{ij}$ will be a simple function of the states of the actually interacting pair $(i,j)$ of automata, i.e., $J_{ij}=f(a_i^t,a_j^t)$, where $a_i^t$

 

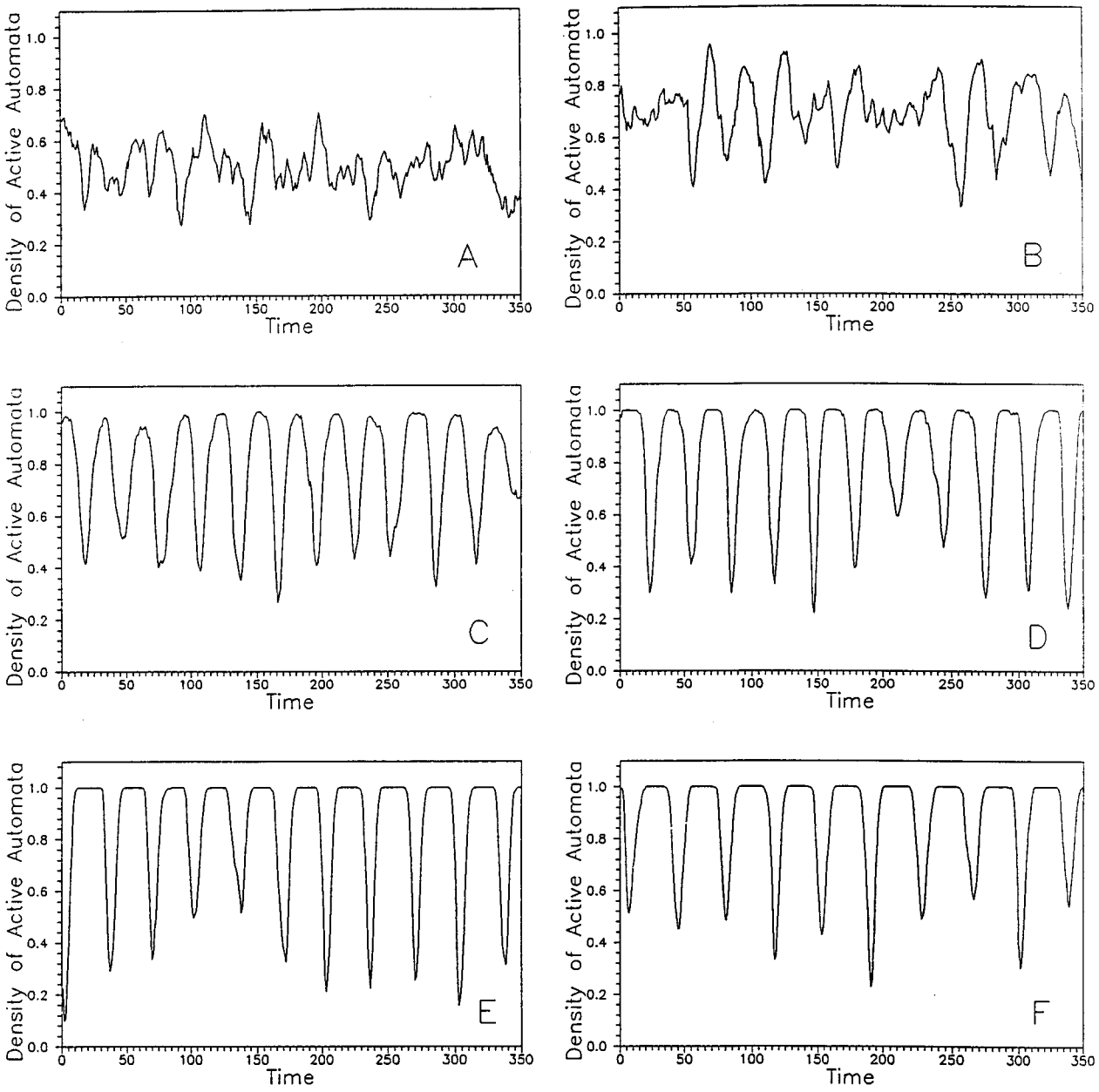


FIG. 1. Temporal behavior of $\rho_t^+$, with parameters $L=50$, $S_a=0.1$, $g=0.1$, $\theta_{\text{act}}=10^{-16}$, $p_a=0.01$, and (a) $\rho=0.10$, (b) $\rho=0.15$, (c) $\rho=0.20$, (d) $\rho=0.25$, (e) $\rho=0.30$, (f) $\rho=0.35$.

$=\Theta(S_i(t)-\theta_{\text{act}})$. In our case, where two basic states are defined the connection matrix reduces to the following $2\times 2$ table:

$$\mathbf{\Lambda}=\begin{bmatrix}\lambda_{11} & \lambda_{10}\\ \lambda_{01} & \lambda_{00}\end{bmatrix}.$$

At a given time step, the interaction $J_{ij}$ between the $i$th and the $j$th elements is equal to $\lambda_{a_i^t a_j^t}\in\mathbf{\Lambda}$ by depending on the activity states of the given elements. More precisely, $J_{ij}$ will be equal to $\lambda_{11}$ when both ants are active, to $\lambda_{10},\lambda_{01}$ when one is active and the other inactive, and to $\lambda_{00}$ if both automata are inactive.

This model is able to account for the oscillations observed in the experiments (see Fig. 1). The model also allows one to define a critical density of active elements, i.e., a density where oscillations appear, that is approximately the same density observed usually in ant colonies (for ''realistic'' sets of parameters, see below) [14]: $\rho_c\simeq 0.2$. Furthermore, recent work has shown that noise is a determinant in the mechanism of oscillations, through spontaneous activation, suggesting that oscillations appear at a noise-induced transition [16] (see [3] for a description of noise-induced transitions). An order parameter for FNNs was also found in [16]: assuming the transition to be noise induced we can define an order parameter by using the stationary density of active elements $P(\rho^+)$ (computed by means of histograms). If we define $\rho_m^+$ such that

$$P(\rho_m^+)=\max_{\rho^+\in[0,1]} P(\rho^+)$$

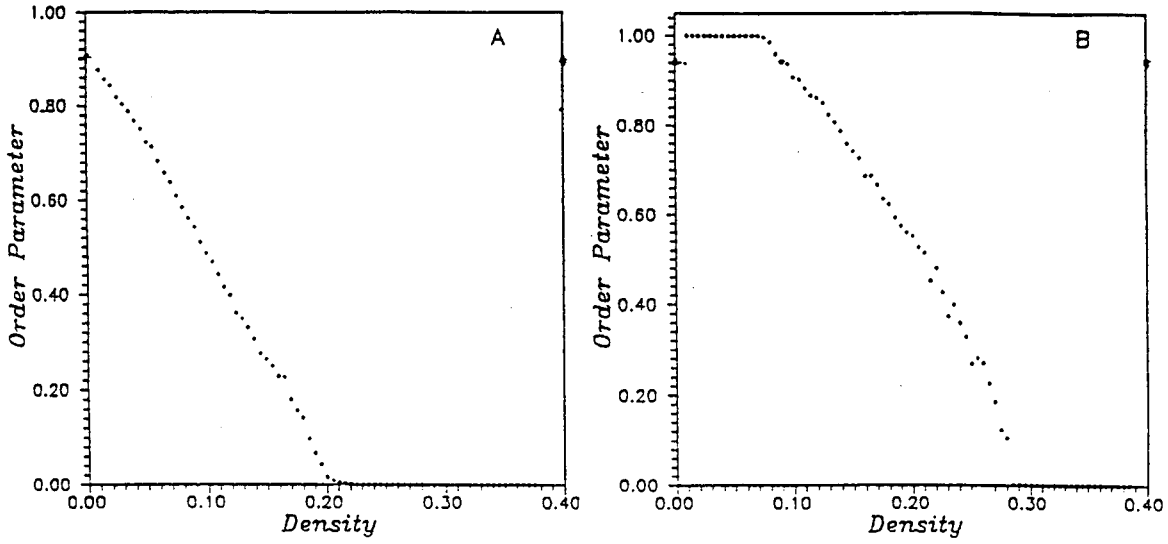


FIG. 2. Order parameter $\Gamma(\rho,p_a)$ with parameters $L=50$, $S_a=0.1$, $g=0.1$, $\theta_{\rm act}=10^{-16}$, and (a) $p_a=0.01$, (b) $p_a=0.001$.

the order parameter will be defined by

$$\Gamma(\rho,p_a)=1-\rho_m^+.$$

As seen in Fig. 2 the value of $\Gamma$ is zero after the transition and nonzero before the transition and, as was shown in [16], the critical density $\rho_c$ is the same as the one that was formerly determined in [14] by means of the Shannon-Kolmogorov entropy.

There has been experimental work measuring some parameters of FNN, such as $J_{ij}$ (interaction between ''neuron ants,'' assumed to be 1 in theoretical simulations) and $g$ (the gain parameter of the nonlinear interaction among individuals, assumed to be 0.1 in the model) [17].

### III. SIMPLE FLUID NEURAL NETWORKS

Some features of the original FNN, as defined in Sec. II, can be considerably simplified. We will define the simple FNN (SFNN) in the following way: We have $N$ individuals $S_i(t)\in\mathbb{R}$ that change their state according to

$$S_i(t+1)=gS_i(t)+g\sum_{j_t^*} J_{ij_t^*}S_{j_t^*}(t)+S_a\Theta(\theta_{\rm act}-S_i(t))I_i^t, \tag{1}$$

where $I_i^t\in\{0,1\}$ with probability $P(I_i^t=1)=p_a$ and we have made a first-order approximation of tanh: $\tanh(x)\simeq x$ removing one of the nonlinearities of the original FNN. The meaning of $J_{ij}$, $S_a$, $p_a$, and $g$ is the same as in the original FNN (Sec. II.). Active states will be defined by $a_i^t=\Theta(S_i(t)-\theta_{\rm act})$.

The meaning of $j_t^*$ is the *neighborhood*. At a given time step $t$, the local field $h_i(t)=\Sigma_{j_t^*}J_{ij_t^*}S_{j_t^*}(t)$ will be computed for all $i$ *before* the change of state $S_i(t+1)$ is performed. In order to do so, *for each individual* $S_i(t)$, $K$ random connections to some individuals will be established (these individuals will be called *the neighbors*). $K$ is chosen randomly from the distribution

$$P(K=k)=\binom{V}{k}\rho^k(1-\rho)^{V-k}. \tag{2}$$

This has the same effect as if we threw, at each time step *and* for each element $S_i(t)$, all the $N$ elements upon an $L\times L$ lattice (then $\rho=N/L^2$ will be the density of elements), in order to compute the corresponding local field $h_i(t)$. Thus, we get some kind of ''annealed'' movement. This is similar to the mean field approximation made in spatially distributed epidemic models [18], where movement was dependent on a parameter $m$ such that the limit $m\to\infty$ was in fact the same as throwing randomly all the elements upon the lattice at each time step. In our case we do so to compute each local field $h_i(t)$, so our system is, in this sense, more disordered.

Considering the density of active individuals at time $t$,

$$\rho_t^+=\frac{1}{N}\sum_{i=1}^{N}\Theta(S_i(t)-\theta_{\rm act}), \tag{3}$$

we can see in Fig. 3 that $\rho_t^+$ in SFNN has the same temporal behavior as $\rho_t^+$ in FNN: irregular behavior at low densities and more ordered oscillatory behavior for growing $\rho$. This allows one to apply the FNN order parameter in this case too. We can see $\Gamma(\rho,p_a)$, as defined in Sec. II, in Fig. 4, computed for a SFNN.

To sum up, we have a simple FNN, where some nonlinearities have been removed and where each individual, at each time step, establishes connections randomly, as if we had some kind of ''annealed'' movement. This has simplified considerably the model without loss of interesting behavior because both FNN and SFNN are phenomenologically identical. In the rest of the paper we will explore the relation between the critical density $\rho_c$ and activation probability $p_a$ in SFNN (as we did numerically with FNN in [16]). The other parameters will be fixed to $g=0.1$, $V=4$, $S_a=0.1$, $J_{ij}=1$ for all $i$, $j$ and $\theta_{\rm act}=10^{-16}$ (see [14] and [16] for a justification of this parameter set).

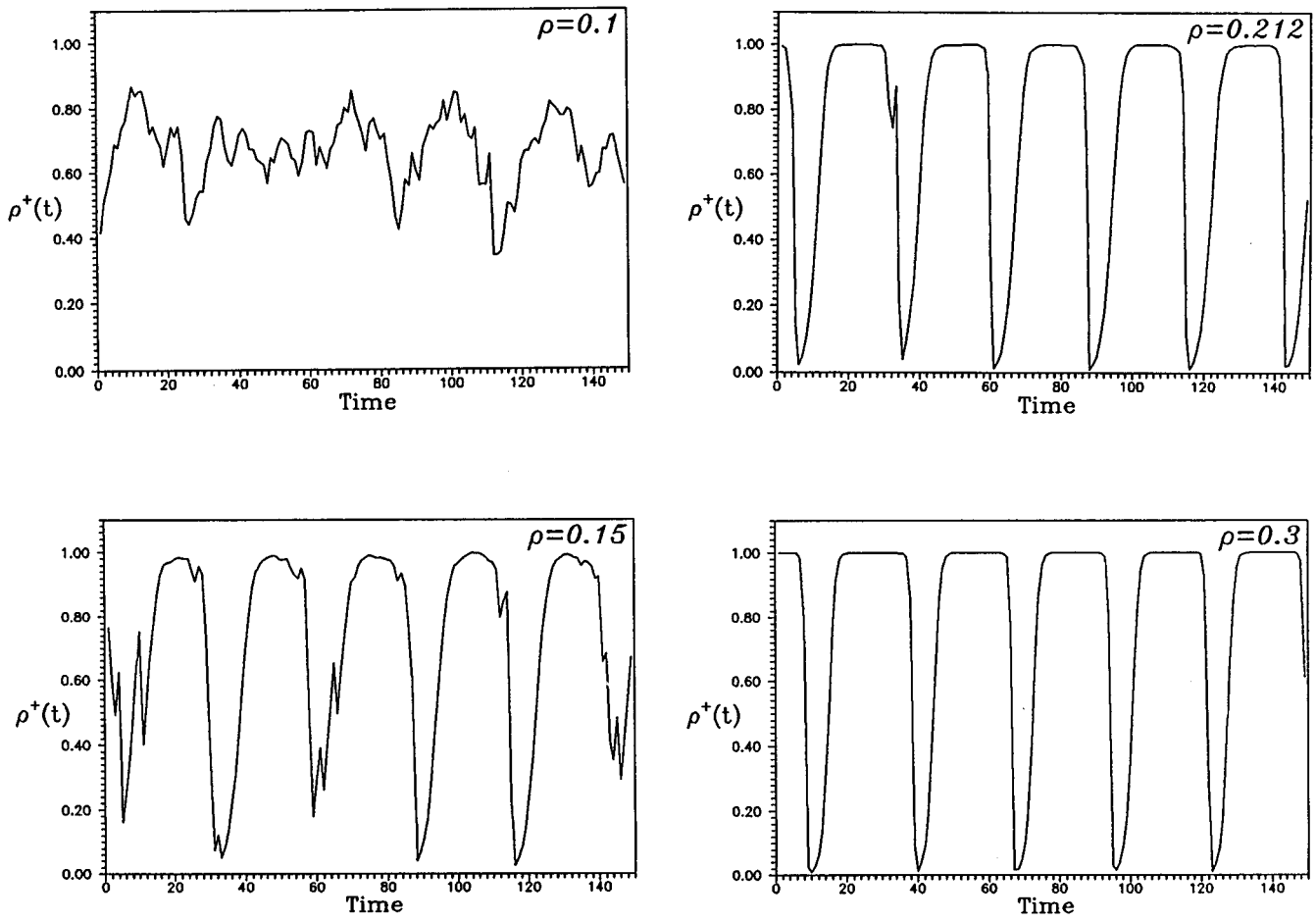


FIG. 3. Temporal behavior of $\rho_t^+$ in SFNN, with parameters $L=100$, $S_a=0.1$, $g=0.1$, $\theta_{\text{act}}=10^{-16}$, $p_a=0.01$, and $V=4$.

## IV. ANALYTIC APPROXIMATION OF $\rho_c$

The analysis will be performed for $\rho>\rho_c$, that is, in the region of well developed oscillations. There the behavior of activity spreading is quite well defined: as we see in Fig. 5 the role of spontaneous activation is merely that of starting

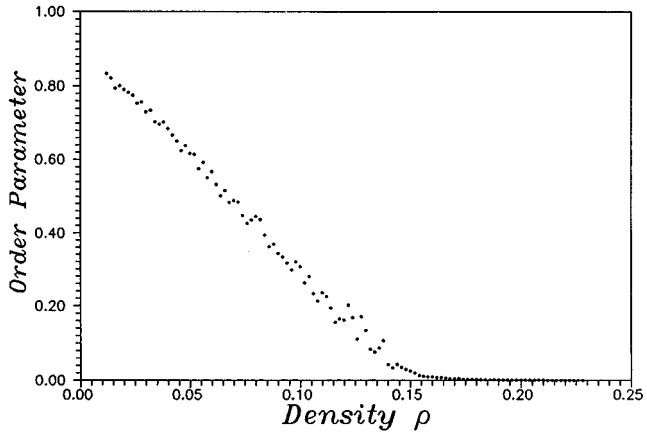


FIG. 4. Order parameter $\Gamma(\rho,p_a)$ in SFNN with parameters $L=100$, $S_a=0.1$, $g=0.1$, $\theta_{\text{act}}=10^{-16}$, $p_a=0.01$, and $V=4$. Each point has been computed with $10^4$ time steps after $2\times10^3$ transitories.

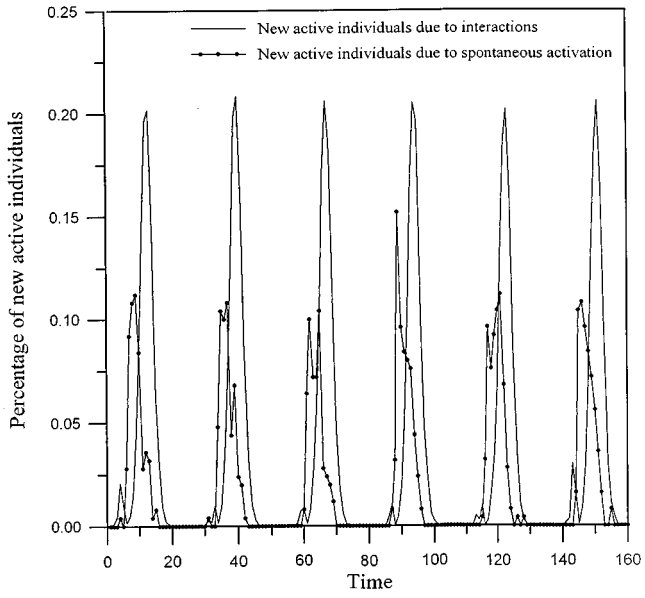


FIG. 5. At each time step the number of *new* active individuals due to interactions and the number of *new* spontaneous active individuals are represented, so new active individuals are classified according to how they became active. *Values in the spontaneous activation curve are multiplied by* 10. Parameters are $L=100$, $S_a=0.1$, $g=0.1$, $\theta_{\text{act}}=10^{-16}$, $p_a=0.01$, $V=4$, and $\rho=0.25$.

the process of activity propagation, a process that continues by means of interaction among individuals until activity reaches the whole system ($\rho_t^+ = 1$). This would allow us to analyze separately activity propagation and inactivation, assuming in both cases that there is no spontaneous activation.

First of all we will find a condition on $V$ and $g$ to assure the decay of the system to a state where all $N$ elements are inactive. With the above mentioned assumption the evolution for $S_i(t)$ will be

$$S_i(t+1) = gS_i(t) + g\sum_{j_t^*} S_{j_t^*}(t).$$

To see the global evolution of the $N$ individuals we can derive a discrete equation for the state average $\langle S(t)\rangle = (1/N)\Sigma_{i=1}^N S_i(t)$ if we approximate the term $\Sigma_{j_t^*} S_{j_t^*}(t)$ by the mean-field version $V\rho\langle S(t)\rangle$ so that

$$\langle S(t)\rangle = [g(1+V\rho)]^t \langle S(0)\rangle$$

and we can assure activation decay if $g(1+V\rho) < 1$. If we assume that $\rho$ is as large as possible ($\rho = 1$), we get the condition

$$g < \frac{1}{1+V},$$

which is satisfied in our case, because $g = 0.1$ and $V = 4$. Of course, if $\langle S(t)\rangle$ tends to 0, $\rho_t^+$ will tend to 0 too. Let us remark that, though $\langle S(t)\rangle$ tends to zero exponentially but smoothly, $\rho_t^+$ goes to 0 in very few time steps (as can be seen in Fig. 3 when $\rho > 0.2$).

Now let us study the propagation of activation through the system. In [16] we put forward the hypothesis that only two factors were important in order to understand FNN oscillations: the average time $\tau'(\rho, p_a)$ one individual is active between two inactive states and activity propagation $Y_{\lfloor Np_a \rfloor}(N, L, p_a)$, that is, the average number of time steps necessary to reach the state of $\rho_t^+ = 1$ from an initial state where $\lfloor Np_a \rfloor$ individuals are active, i.e., the mean (integer) number of individuals that would active spontaneously with probability $p_a$ in a system with all $N$ elements inactive. These are precisely the factors we will analyze in order to compute analytically $\rho_c$. Intuitively, if $Y_{\lfloor Np_a \rfloor}(N, L, p_a)$ is less than $\tau'(\rho, p_a)$ the state of maximum activation will be reached *before* individuals start the process of inactivation, then we will observe oscillations. So then, $\rho_c$ will be such that

$$Y_{\lfloor Np_a \rfloor} = \tau'. \tag{4}$$

Activity spreading can be treated as a branching process if when considering activity by interaction we take into account only the state $a_i^t$ of each individual. In this way, we will say that an inactive individual is activated by its neighbors if there is at least one of them active [it is obvious that this is not the *exact* mechanism by which individuals activate each other, since an individual with all neighbors active, each one with a very small $S_i(t)$, might not be activated]. The probability of having at least an active individual as a neighbor is easy to compute because of the ''annealed movement'' we have introduced. If we have $i$ active individuals, the above-mentioned probability is

$$\gamma_i = 1 - \left(1 - \frac{i}{L^2}\right)^V. \tag{5}$$

As we are only considering activity spreading in the oscillation phase, there will be no activity decay, allowing us to compute the probability of $j$ active individuals having $i$ individuals active in the previous time step

$$P_{ij} = P(A_{t+1} = j | A_t = i) = \begin{cases} \binom{N-i}{j-i} \gamma_i^{j-i} (1-\gamma_i)^{N-j}, & \text{if } i \leqslant j, \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $A_t$ is the number of active individuals at time $t$. This defines a branching process that will finish when $A = N$. We will treat this process as a Markov chain [19] with the stochastic matrix

$$P = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1N} \\ 0 & P_{22} & \cdots & P_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_{NN} \end{pmatrix} \tag{7}$$

with which we can compute the mean number of steps before being absorbed by the unique closed class of our system, the one element set $\{N\}$. In order to perform the calculations, the $P$ matrix has to be rearranged to get the canonical form

$$P^* = \begin{pmatrix} P_1 & 0 \\ R & Q \end{pmatrix} = \begin{pmatrix} P_{NN} & 0 & \cdots & 0 \\ P_{1N} & P_{11} & \cdots & P_{1(N-1)} \\ P_{2N} & 0 & \cdots & P_{2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ P_{(N-1)N} & 0 & \cdots & P_{(N-1)(N-1)} \end{pmatrix} \tag{8}$$

so that the *fundamental matrix* $M = [I - Q]^{-1}$ of the Markov chain can be found. The matrix $M$ plays a central role in transient analysis of Markov chains [19]. $M$ gives immediately the quantity we want to compute. It is easy to verify that the $ij$th element of $Q^k$, $q_{ij}^{(k)}$, is the probability of a transition from the state $A = i$ to the state $A = j$ in exactly $k$ steps. The average number of times that, starting in state $A = i$, the process reaches state $A = j$ before it leaves transient states and enters the closed class is

$$q_{ij}^{(0)} + q_{ij}^{(1)} + q_{ij}^{(2)} + \cdots + q_{ij}^{(k)} + \cdots$$

that is, precisely $M_{ij}$, since the identity

$$M = [I - Q]^{-1} = I + Q + Q^2 + \cdots + Q^k + \cdots$$

follows from the fact that $Q$ has all the eigenvalues strictly inside the unit circle (the eigenvalues of $Q$ are $\lambda_j = P_{jj}$ for $1 \leqslant j \leqslant N-1$ and $\lambda_j < 1$ since $P$ is a stochastic matrix). If $\mathbf{1}$ is a column vector whose components are all equal to 1, the mean number of steps before reaching the state of all indi-

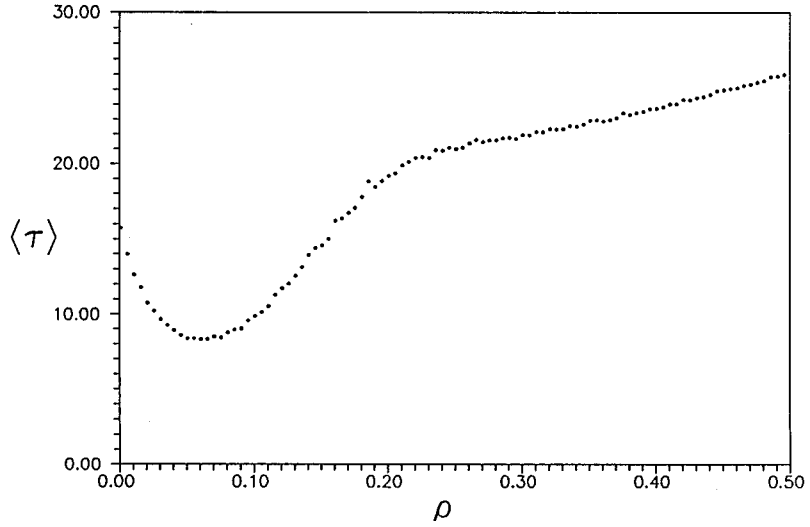


FIG. 6. The mean time an individual is active between two inactivations. We can see that $\langle\tau\rangle$ is density dependent with a clear change in the shape of the curve around $\rho_c$ ($\rho_c \simeq 0.212$ in this case). Parameters are $L=100$, $S_a=0.1$, $g=0.1$, $\theta_{\text{act}}=10^{-16}$, $p_a=0.01$, and $V=4$. $\langle\tau\rangle$ is computed from $10^3$ time steps after $10^3$ transitories.

viduals active, taking as a departure point a state $A=i$, is the $i$th component of the vector $M\mathbf{1}$. So, if $Y_j=(M\mathbf{1})_j$, solving the linear system

$$M^{-1}\mathbf{Y}=(I-Q)\mathbf{Y}=\mathbf{1}$$

we can compute any $Y_j$ with the recurrence

$$Y_j=(1-P_{jj})^{-1}\left(1+\sum_{i=j+1}^{N-1} P_{ji}Y_i\right), \tag{9}$$

which can be computed easily, due to the fact that $P_{ij}=0$ if $j<i$.

Now that we have $Y_{\lfloor Np_a\rfloor}(N,L,p_a)$ we need to compute $\tau'(\rho,p_a)$. This is a rather difficult calculation and we have approximated $\tau'(\rho,p_a)$ by the inactivation time of a solitary individual subject to simple perturbations

$$S_{t+1}=gS_t+\xi_t, \tag{10}$$

where $\xi_t$ are independent identically distributed (IID) random variables. These random variables will be such that we should assure that $\langle S_t\rangle \to \theta_{\text{act}}$, so we will impose $\langle\xi_t\rangle=(1-\epsilon)\theta_{\text{act}}$ where $\epsilon$ is a free parameter whose value will be established below. The evolution of $\langle S_t\rangle$ is easily found (with $S_0=S_a$) from

$$S_t=g^tS_a+\sum_{j=1}^{t} g^{t-j}\xi_j$$

and it is

$$\langle S_t\rangle=g^tS_a+(1-\epsilon)\theta_{\text{act}}G_t, \tag{11}$$

where

$$G_t=\sum_{j=0}^{t-1} g^j.$$

Since $g=10^{-1}$, $G_t$ is easily calculated and it is quite obvious that it can be approximated by $G_\infty$. So then, our estimation of $\tau'(\rho,p_a)$, $\tau_{\text{per}}$, will be given by

$$\tau_{\text{per}}=(\log_{10} g)^{-1}\{\log_{10}\theta_{\text{act}}-\log_{10}S_a + \log_{10}[1-G_\infty(1-\epsilon)]\}. \tag{12}$$

From Eq. (12) it is clear that $\epsilon$ is bounded by

$$\epsilon>1-\frac{1}{G_\infty},$$

that is, $\epsilon>0.1$ and by $\epsilon<1$, because if $\epsilon=1$ then $\langle S_t\rangle=g^tS_a$ and

$$\tau_{\text{per}}=\frac{\log_{10}(\theta_{\text{act}}/S_a)}{\log_{10}(g)},$$

that is, $\tau_{\text{per}}=15$ for the set of parameters we are working with.

But how do we determine exactly the value of $\epsilon$? At this point we must resort to the numerically computed $\tau'(\rho,p_a)$ for the SFNN (see Fig. 6). Let $\delta_\epsilon=\epsilon-0.1$. From Eq. (12) we see that $\tau_{\text{per}}$ gets larger as $\delta_\epsilon$ gets smaller, but, looking at Fig. 6, particularly at the region around $\rho_c$, we see that a $\tau_{\text{per}}$ much larger than 15 does not make sense. Therefore, we will fix $\delta_\epsilon=0.01$, a value large enough to keep $\tau_{\text{per}}$ in the ''meaningful'' region and small enough to make $\tau_{\text{per}}>15$. Finally, with the set of parameters we have been using and the $\delta_\epsilon$ mentioned above, $\tau_{\text{per}}\simeq 16.95$.

Once we have $Y_{\lfloor Np_a\rfloor}(N,L,p_a)$ and $\tau'(\rho,p_a)\simeq\tau_{\text{per}}$ we can find a density $\rho_c$ such that $Y_{\lfloor Np_a\rfloor}\simeq\tau_{\text{per}}$. This analytically computed $\rho_c^{\text{an}}$ is compared with a numerically determined $\rho_c^{\text{nu}}$ using the order parameter $\Gamma(\rho,p_a)$, in Fig. 7. $\rho_c^{\text{an}}$ and $\rho_c^{\text{nu}}$ both have a linear dependence on $\log_{10}(p_a)$ and agree accurately.

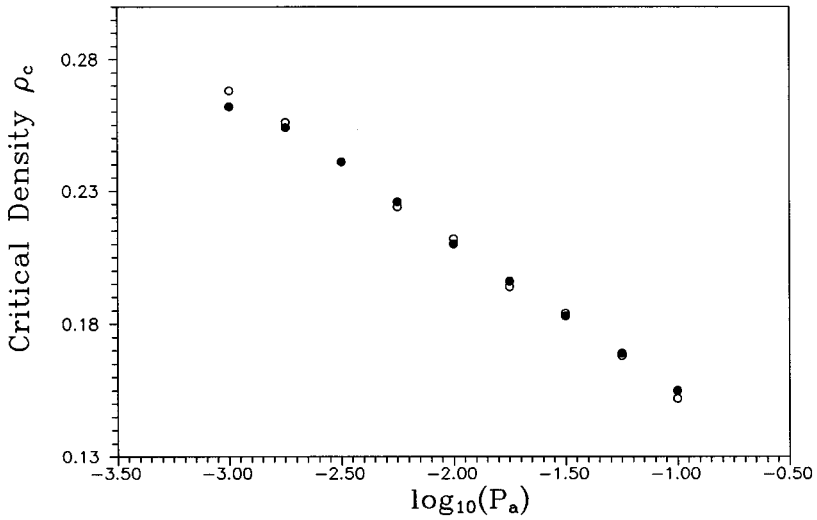


FIG. 7. $\rho_c^{\mathrm{nu}}$ (open circles) and $\rho_c^{\mathrm{an}}$ (solid circles) as a function of $\log_{10}(p_a)$ Parameters are $L=100$, $S_a=0.1$, $g=0.1$, $\theta_{\mathrm{act}}=10^{-16}$, and $V=4$. $\rho_c^{\mathrm{nu}}$ is computed from $\Gamma(\rho,p_a)$ with $10^4$ time steps after $2\times10^3$ transitories, averaged over 10 samples.

## V. DISCUSSION

In this paper we have explored the emergence of oscillatory behavior in a mean-field model of FNNs. FNNs have been successfully used as models of collective behavior, from oscillations [11,14] to the problem of universal computation [20]. The introduction of mobility by simple units enables us to go beyond the classical models based on excitable systems [21] though it also makes it difficult to reach analytic results. Here, following a mean-field approach to the original FNN with sigmoidal response, we have derived a simple analytic result that provides a good understanding for the origin of oscillatory behavior and the existence of noise-induced transitions.

Some simplifications on the FNN model have allowed us to define the SFNN, with identical phenomenology but with the possibility of performing analytical work. Working on the $\rho>\rho_c$ phase we have assumed no spontaneous activation and we have analyzed separately the inactivation process and the activation process, that is, the two parts that compose one oscillation in the $\rho_t^+$ curve. We have derived a relation between $g$ and $V$ in order to assure inactivation and we have checked a hypothesis put forward in [16] in order to approximate $\rho_c$. This hypothesis and approximations of $Y_{\lfloor Np_a\rfloor}(N,L,p_a)$ and $\tau'(\rho,p_a)\simeq\tau_{\mathrm{per}}$ provide a $\rho_c^{\mathrm{an}}$ value that agrees with $\rho_c^{\mathrm{nu}}$ [obtained from $\Gamma(\rho,p_a)$]. Work in progress is focusing on a better analytical approximation of $\tau'(\rho,p_a)$ through the inclusion of interactions among individuals, since the perturbation approach, which needs the introduction of a parameter $\epsilon$ whose value has to be found heuristically, is not entirely satisfactory.

Finally let us note that our analytic work makes clear that it does not make sense to consider the large $N$ and $L$ cases, since our system depends on the mean time of activity propagation $Y_{\lfloor Np_a\rfloor}(N,L,p_a)$, which grows with $N$ and $L$ so in a ''thermodynamic limit'' it will diverge, and on $\tau'(\rho,p_a)$ that is density dependent (see Fig. 6). Therefore, in the limit of large $N$ and $L$ there will be no oscillations, which have been observed in both the FNN and the SFNN [22]. Then, in what sense do we refer to a ''mean-field'' theory? The point here is the *annealed movement* of the individuals that decorrelates completely their interactions [18]. This is a mean-field approximation, despite its departure from the behavior of mean-field theories in equilibrium systems [23].

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[1] M. M. Millonas, J. Theor. Biol. **159**, 529 (1992).
[2] B. J. Cole, Am. Nat. **137**, 244 (1991).
[3] W. Horsthemke and R. Lefever, *Noise-Induced Transitions* (Springer-Verlag, Berlin, 1984).
[4] E. Bonabeau, J. Phys. I **6**, 309 (1996). This is a particularly well known example of how cooperative phenomena close to instability points lead to flexible and efficient behavior by ant colonies. Considered as an $N$-body problem involving nonlinear autocatalytic responses, it can be shown that the swarm as a whole exhibits phase transitions that are described by a field model $\partial_t\phi=(\mu-\mu_c)\phi-\phi^3$. The predicted bifurcations are consistent with the experiments. As in our study the effect of noise is very important, although in the context of classical second order phase transitions.
[5] B. Hölldobler and E. O. Wilson, *The Ants* (Springer-Verlag, Berlin, 1990).
[6] *Intelligence Collective*, edited by E. Bonabeau and G. Theraulaz (Hermes, Paris, 1994).
[7] E. O. Wilson, *The Insect Societies* (The Belknap Press of Harvard University Press, Cambridge, MA, 1971).
[8] N. Franks, S. Bryant, R. Griffiths, and L. Hemerik, Bull. Math. Biol. **52**, 597 (1990).

[9] L. Hemerik, N. F. Britton, and N. R. Franks, Bull. Math. Biol. **52**, 613 (1990).
[10] C. Tofts, M. Hatcher, and N. R. Franks, J. Theor. Biol. **157**, 71 (1992).
[11] R. V. Solé, O. Miramontes, and B. C. Goodwin, J. Theor. Biol. **161**, 343 (1993).
[12] M. J. Hatcher, C. Tofts, and N. R. Franks, Naturwissenschaften **79**, 32 (1992).
[13] G. E. Robinson, Annu. Rev. Entomol. **37**, 637 (1992).
[14] R. V. Solé and O. Miramontes, Physica D **80**, 171 (1995).
[15] D. Amit, *Modeling Brain Function* (Cambridge University Press, Cambridge, 1989).
[16] J. Delgado and R. V. Solé, Phys. Lett. A **229**, 183 (1997).
[17] B. J. Cole and D. Cheshire, Am. Nat. **148**, 1 (1996).
[18] N. Boccara, C. Kyeong, and M. Oram, J. Phys. A **27**, 1585 (1994).
[19] D. G. Luenberger, *Introduction to Dynamic Systems* (John Wiley & Sons, New York, 1979).
[20] R. V. Solé and J. Delgado, Complexity **2**, 49 (1996).
[21] A. S. Mikhailov, *Foundations of Synergetics I* (Springer-Verlag, Berlin, 1990).
[22] J. Delgado and R. V. Solé (unpublished).
[23] J. J. Binney, N. J. Dowrick, A. J. Fisher, and M. E. J. Newman, *The Theory of Critical Phenomena: An Introduction to the Renormalization Group* (Oxford University Press, Oxford, 1992), Chap. 6.